Adaptive Algorithm-based DFE for Low PAPR Consumption and Effective Bandwidth Utilization of MIMO LTE OFDM

Sirajum Munira, A.K.M. Fazlul Haque

Abstract— This paper focused on low PAPR consumption and efficient bandwidth optimization for MIMO LTE Downlink OFDM system excluding cyclic prefix (CP). The FDADFE is subjected to extract the performance based on specific parameters for better BER characteristics. The 3GPP LTE employs OFDM in downlink transmission, which experiences relentless inter-symbol interference (ISI) in high frequency selective fading channels. Although cyclic prefix (CP) is employed in the OFDM symbols to eliminate ISI completely, it consumes bandwidth and power. In this work, a frequency domain adaptive decision feedback equalizer (FDADFE) is introduced. The equalizer applies both RLS and LMS algorithms to adapt the filter coefficient that makes the system more convergent executing less time. Although complexity grows in the proposed system, the simulation results prove that, the FDADFE with OFDM system excluding CP in LTE outperforms the conventional OFDM system including CP in LTE in terms of both power and bandwidth efficiency with better BER performance.

Index Terms—CP, BER, Bandwidth, DFE, RLS, LMS.

1 INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) is a promising multicarrier modulation technique which comprises vitality to the multipath fading channels, low complexity of implementation, higher spectral efficiency and also the capability to deliver flexible transmission bandwidths with inventive features such as frequency selective fading, multiple input multiple output (MIMO), and interference coordination. In the 3GPP LTE standard, the downlink transmission is based on an OFDM scheme, which represents the broadband transmission bandwidth as a collection of several narrowband subchannels. However, in OFDM system, a cyclic extension which is called Cyclic Prefix (CP) is prepended with every OFDM symbols to combat multipath fading. Although employing CP completely eliminates ISI resulting in multipath propagations, it also consumes power and bandwidth, which is very significant problem for LTE. The frequency domain Decision Feedback Equalizer (DFE) can outperform most in such a case. But it is mostly applied in Single-Carrier Frequency Division Multiple Access (SC-FDMA) including CP in the uplink direction of LTE. Most of them used non adaptive algorithms for equalization to perform. So, in this paper, a competent Frequency-Domain Adaptive Decision feedback Equalizer (FDADFE) is proposed for MIMO LTE OFDM system. A DFE with an adaptive algorithm system with feed-forward and feedback filters is developed both of which operates in the frequency domain. Due to the fast tracking/convergence properties, the Recursive Least Squares (RLS) algorithm is used to update the set of weights of the feed-forward and feedback filters; however due to its computational complexity, it executes slowly[8]. Hence the next iteration shifts to the most popular Least Mean Square (LMS) algorithm due to its fast execution properties [8].

A number of research articles are established in the literature that proposes the most possible ways to condense, or even completely abolish the noise, by using more complicated receiver signal processing in an OFDM transmission system. Most of the works have been developed with the Single-Carrier Frequency Division Multiple Access (SC-FDMA) in the uplink transmission system. In a work, a Hybrid-DFE and iterative block decision feedback equalization are considered to improve the equalization performance of SC-FDMA, where the feed-forward filer is implemented in the frequency domain while the feedback filter is realized in time domain [2]. Therefore, due to the frequency domain feed-forward filter, the complexity has been reduced significantly comparing to its time domain equivalent. In another work, an effective MIMO MMSE DFE has been proposed for OFDM system without CP for IEEE802.11 (a), in which effective bandwidth utilization and channel interpolation was performed [3]. All of these above mentioned DFEs are non-adaptive and operated in the frequency domain. In another work, a low complexity Adaptive Frequency-Domain Decision Feedback Equalizer (AFD-DFE) is introduced, where both the feed-forward and feedback filters operate in the frequency domain, and the weights are adapted by means of the block Recursive Least Squares (RLS)

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algorithm for uplink SC-FDMA[4][5]. In a very recent work, an adaptive DFE has been introduced in LTE downlink OFDM transmission including CP, in which BER performance was evaluated [10].

The scheme proposed by Alamouti is a special case of space-time block codes (STBCs) [5] [7]. Alamouti's STBC can be applied to the 3rd Generation Partnership Project (3GPP) LTE downlink over two OFDM symbols and two transmit antennas [1].

Following this introduction, the organization of this paper is devoted to Literature Review in Section 2. In Section 3, the research model of this paper is presented. The development of the FDADFE for LTE MIMO OFDM is shown in section 4. The performance and simulation results are discussed in Section 5. Finally, Section 6 draws the conclusions of the work.

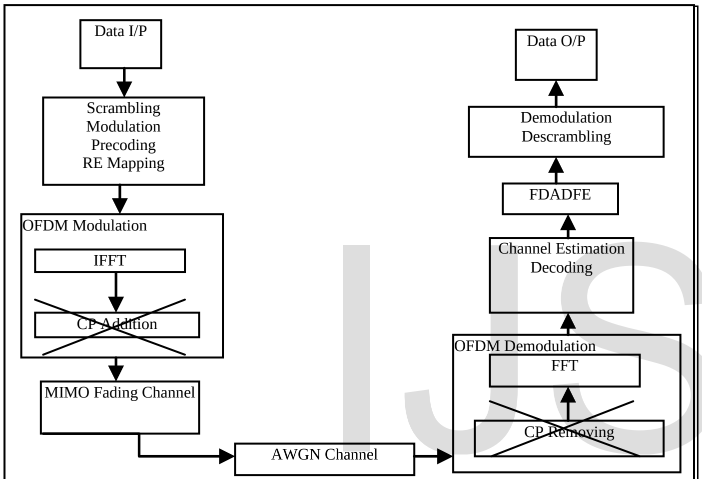

Fig.1. Block diagram of an OFDM transmission system excluding CP in 3GPP LTE MIMO with Transmit Diversity

2 System Model

All through this paper, the system model applied is shown in the Fig. 1. At first, a set of binary data is generated as a payload. Modulation is followed by the scrambling of the payload. The LTE employs three types of different modulation techniques in three different cases.

At the transmitter, the data block is represented after scrambling and modulation, as

$\chi^{(m)}$=[X(0)$^{(m)}$, X(1)$^{(m)}$, …, X(M-1)$^{(m)}$]^T. Using (7), we get

$\chi_1^{(m)}$=[X(0)$^{(m)}$,-X*(1)$^{(m)}$,… X(M-2)$^{(m)}$,-X*(M-1)$^{(m)}$]^T and

$\chi_2^{(m)}$=[X(1)$^{(m)}$,X*(0)$^{(m)}$,… X(M-1)$^{(m)}$, X*(M-2)$^{(m)}$]^T,

where (.)* denotes the complex conjugate operation. After mapping the resource elements for LTE downlink applying the N-point IFFT, the transmitted signals are denoted by $\varkappa_1^{(m)}$ and $\varkappa_2^{(m)}$ corresponding to $\chi_1^{(m)}$ and $\chi_2^{(m)}$. Here, including CP is avoided in the fig.1 by applying X on the blocks.

The received signal for the m_{th} user, after SFBC [9], can be expressed as

$$\boldsymbol{y}_{oe}=\begin{bmatrix} y_o \\ y_e^* \end{bmatrix}=\begin{bmatrix} \boldsymbol{\Lambda}_{10} & \boldsymbol{\Lambda}_{20} \\ \boldsymbol{\Lambda}_{2e}^* & -\boldsymbol{\Lambda}_{1e}^* \end{bmatrix}\begin{bmatrix} \boldsymbol{\chi}_o \\ \boldsymbol{\chi}_{oe} \end{bmatrix}+\begin{bmatrix} N_o \\ N_e^* \end{bmatrix} \quad (1)$$

$$\triangleq \boldsymbol{\Lambda\chi}_{oe} + N_{oe} \quad (2)$$

Where $y_o(\boldsymbol{\chi}_o)$and $y_e(\boldsymbol{\chi}_e)$ represents the odd ad even components, respectively, of the frequency domain received signal $y(\boldsymbol{\chi})$. $\boldsymbol{\Lambda}_{io}$ and $\boldsymbol{\Lambda}_{ie}$ are the diagonal matrices which contain odd and even components, respectively, of the frequency-domain channel corresponding to the ith transmit antenna. We assumed,

$\boldsymbol{\Lambda}_{ie} = \boldsymbol{\Lambda}_{io}$, i=1, 2

After MMSE, we get

$$\begin{bmatrix} \widehat{\mathbf{X}}_o \\ \widehat{\mathbf{X}}_e \end{bmatrix}=\left(\boldsymbol{\Lambda}^H\boldsymbol{\Lambda}+\frac{1}{SNR}\mathbf{I}_{2M}\right)^{-1}\boldsymbol{\Lambda}^H\, \boldsymbol{y}_{oe} \quad (3)$$

where SNR is the signal-to-noise ratio at the receiver. Since $\Lambda^H\Lambda$ has an Alamouti-like structure, therefore

$$\begin{bmatrix} \widehat{\mathbf{X}}_o \\ \widehat{\mathbf{X}}_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 \\ \boldsymbol{\varphi}_2^* & -\boldsymbol{\varphi}_1^* \end{bmatrix}\begin{bmatrix} y_o \\ y_e^* \end{bmatrix} \quad (4)$$

where $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$ are diagonal matrices. Alternatively, it can be written as

$$\begin{bmatrix} \widehat{\mathbf{X}}_o \\ \widehat{\mathbf{X}}_e \end{bmatrix} = \begin{bmatrix} \mathrm{diag}(y_o) & \mathrm{diag}(y_e^*) \\ -\mathrm{diag}(y_e) & \mathrm{diag}(y_o^*) \end{bmatrix}\begin{bmatrix} \boldsymbol{\Upsilon}_1 \\ \boldsymbol{\Upsilon}_2 \end{bmatrix} \quad (5)$$

where $\boldsymbol{\Upsilon}_1$ and $\boldsymbol{\Upsilon}_2$ are the vectors which contain the diagonal elements of $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$. For a DFE, we have

$$\widehat{\mathbf{X}}_{oe}=\begin{bmatrix} \widehat{\mathbf{X}}_o \\ \widehat{\mathbf{X}}_e \end{bmatrix} = \begin{bmatrix} \mathrm{diag}(y_o) & \mathrm{diag}(y_e^*) \\ -\mathrm{diag}(y_e) & \mathrm{diag}(y_o^*) \end{bmatrix}\begin{bmatrix} \boldsymbol{\Upsilon}_1 \\ \boldsymbol{\Upsilon}_2 \end{bmatrix}+ \begin{bmatrix} \mathrm{diag}(\boldsymbol{\mathcal{D}}_o) & 0 \\ 0 & \mathrm{diag}(\boldsymbol{\mathcal{D}}_e^*) \end{bmatrix}\begin{bmatrix} \boldsymbol{\psi}_1 \\ \boldsymbol{\psi}_2 \end{bmatrix}$$

$$\triangleq \boldsymbol{\mathcal{Z}\mathcal{F}} + \boldsymbol{\mathcal{D}\mathcal{B}} \quad (6)$$

where $\boldsymbol{\mathcal{D}}_o$ and $\boldsymbol{\mathcal{D}}_e$ are $\boldsymbol{\chi}_o$ and $\boldsymbol{\chi}_e$, for the training mode and decision-directed mode, respectively, on $\boldsymbol{\chi}_o$ and $\boldsymbol{\chi}_e$. $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{B}}$ represents the feed-forward filter and feed-back filter co-efficient in the frequency domain. The contain elements $\{\boldsymbol{\Upsilon}_1, \boldsymbol{\Upsilon}_2\}$ and $\{\boldsymbol{\psi}_1, \boldsymbol{\psi}_2\}$, respectively. $\boldsymbol{\mathcal{Z}}$ is an MxM matrix containing received symbols and $\boldsymbol{\mathcal{D}}$ is a diagonal matrix containing the decisions. These co-efficient are computed adaptively using the RLS and LMS algorithm.

At the k_{th} instant, the output of the equalizer can be given as

$$\widehat{\mathbf{X}}_{oe,k}= \boldsymbol{\mathcal{Z}}_k\boldsymbol{\mathcal{F}}_{k-1} + \boldsymbol{\mathcal{D}}_k\boldsymbol{\mathcal{B}}_{k-1} \quad (7)$$

3 FDADFE

3.1 RLS Update

The mean square error (MSE) at the i_{th} frequency is given by

$$MSE(i)=E\left|\mathbf{D}(i) - \widehat{\mathbf{X}}_{oe}(i)\right|^2 \quad (8)$$

where E[.] represents the expectation operation. Minimizing (8) for the feed-forward filter and the feedback filters separately results in the following updates

$$\mathcal{F}_k(i) = \mathcal{F}_{k-1}(i) + \frac{\mu_k}{\epsilon_k + E|y(i) * y(i)|} y_k^*(i) \times \{D_k(i) - [y_k(i)\mathcal{F}_{k-1}(i) + D_k(i)\mathcal{B}_{k-1}(i)]\} \quad (9)$$

$$\mathcal{B}_k(i) = \mathcal{B}_{k-1}(i) + \frac{\mu_k}{\epsilon_k + E|\mathcal{D}(i) * \mathcal{D}(i)|} \mathcal{D}_k^*(i) \times \{D_k(i) - [y_k(i)\mathcal{F}_{k-1}(i) + D_k(i)\mathcal{B}_{k-1}(i)]\} \quad (10)$$

Next, $E|\mathcal{Y}(i) * \mathcal{Y}(i)|$ and $E|\mathcal{D}(i) * \mathcal{D}(i)|$ are replaced by their estimates, which are chosen to be the exponentially weighted sample averages for some scalar $0 << \lambda \leq 1$. Choosing the step size as $\mu_k = 1/(k+1)$ and the regularization factor as $\epsilon_k = \frac{\lambda^{k+1}\epsilon}{k+1}$.

Using the same approach as in [5], we have

$$\mathcal{P}_k^1 = \lambda^{-1}[\mathcal{P}_{k-1}^1 - \lambda^{-1}\mathcal{P}_{k-1k}^1 \psi_k^1 \mathcal{P}_{k-1}^1] \quad (11)$$

$$\mathcal{P}_k^2 = \lambda^{-1}[\mathcal{P}_{k-1}^2 - \lambda^{-1}\mathcal{P}_{k-1}^2 \psi_k^2 \mathcal{P}_{k-1}^2] \quad (12)$$

where λ is the forgetting factor chosen close to 1 and $\psi_k^2 = (|\mathcal{D}_k|^{-2} + \lambda^{-1}\mathcal{P}_{k-1}^2)^{-1}$. However, matrix $\mathcal{P}_k$ has a diagonal structure, i.e., $\mathcal{P}_k = \text{diag}([\mathcal{P}_k^1 \ \mathcal{P}_k^2])$, where $\mathcal{P}_k^1$ and $\mathcal{P}_k^2$ are diagonal as well. Finally, collecting all the coefficients in one vector $\mathcal{W}$, the RLS recursion has the form

$$\mathcal{W}_k = \mathcal{W}_{k-1} + \text{diag}([\mathcal{P}_k^1 \ \mathcal{P}_k^2]) \begin{bmatrix} \mathcal{Z}_k & 0 \\ 0 & \mathcal{D}_k \end{bmatrix}^H \begin{bmatrix} \mathcal{D}_k - \hat{\mathbf{X}}_k \\ \mathcal{D}_k - \hat{\mathbf{X}}_k \end{bmatrix} \quad (13)$$

where $\mathcal{D}_\mathbf{k}$ denotes the decisions at the k_{th} instant, i.e., $\mathcal{D}_\mathbf{k} = \begin{bmatrix} \mathcal{D}_{o,k} \\ \mathcal{D}_{e,k}^* \end{bmatrix}$.

3.2 LMS Update

After updating the weight by the RLS algorithm, the LMS algorithm executes fast with the updated weights. The LMS algorithm assumes $\mathcal{W}_k$ as the previous weight and therefore

$$\mathcal{W}_{k+1} = \mathcal{W}_k + 2\mu e_k \hat{\mathbf{X}}_{oe,k} \quad (14)$$

where μ controls stability and the rate of convergence and

$e_k = \mathcal{Y}_{oe,k} - \mathcal{W}_k^T \hat{\mathbf{X}}_{oe,k}$

The weights of the LMS algorithm improve fast as the weights are adjusted and the filter learns the signal characteristics [10]. Eventually, the weights converge. The condition for convergence is set as $0 < \mu < 1/\lambda_{max}$.

4 Simulation Results

In this paper, the performance of FDAFE is evaluated by computer simulations using LTE Downlink OFDM for PDSCH only. MATLAB14(a) is used for all the simulations presented in this dissertation. The work is done in the LTE PHY in MATLAB for downlink transmission i.e., the system consists of three sections, namely the transmitter, the channel and the receiver, and each section consists of a number of blocks. The system is designed for 2x2 antenna configuration with Transmit Diversity.

To evaluate the performance of the FDADFE with LTE MIMO OFDM system excluding CP and compare it to the conventional LTE systems, two evaluation parameters were chosen. The first one is the bit error rate (BER) and the second one is the peak to average power ratio (PAPR).

The simulation parameters are tabulated below:

TABLE 1

Simulation Parameters

Parameters	Values
Bandwidth, BW	20MHz
Modulation	QPSK, 16QAM, 64QAM
Total No. of Subcarriers, N	2,048
Total No. of Resource Block, Nrb	100
Channel sampling rate	30.72MHz
CP	Excluded
Control Register	2
No. of Transmitter	2
No. of Receiver	2
Subcarrier bandwidth	15kHz
No. of Resource Block per subcarrier	12
No. of symbol per subframe	7
Total No. of Resources	16,800
Total number of data bits	91,200
No. of subframe per frame	2
Transmission Mode of MIMO	Transmit Diversity
FDADFE	Used

TABLE 2

FDADFE Parameters

FDADFE parameters	Values
No. of feed-forward filters	N_f=10
No. of feed-back filters	N_b=5
No. of algorithms used	2

According to the simulation scenario described above, the simulation results are presented and analyzed in this section. The simulation has been performed with the communication system toolbox. Our approach for this paper and analysis is to investigate the BER with effective bandwidth utilization and PAPR performance of FDADFE for 3GPP LTE OFDM system excluding CP with the adaptive MIMO technique as transmit Diversity and compare the results with conventional LTE OFDM including CP as well as theoretical system for three different types of modulation, QPSK, 16QAM and 64QAM.

4.1 BER improvement and Efficient Bandwidth Utilization with FDADFE for LTE MIMO OFDM system excluding CP

This subsection analyses performance of FDADFE for LTE OFDM system excluding CP vs. conventional system including CP. Adding CP is the important part of the OFDM technique to remove ISI and ICI. The size of the CP is usually chosen as 25% of the data. This actually consumes both bandwidth and power. In this simulation, addition of CP is avoided to optimize the bandwidth in its maximum level.

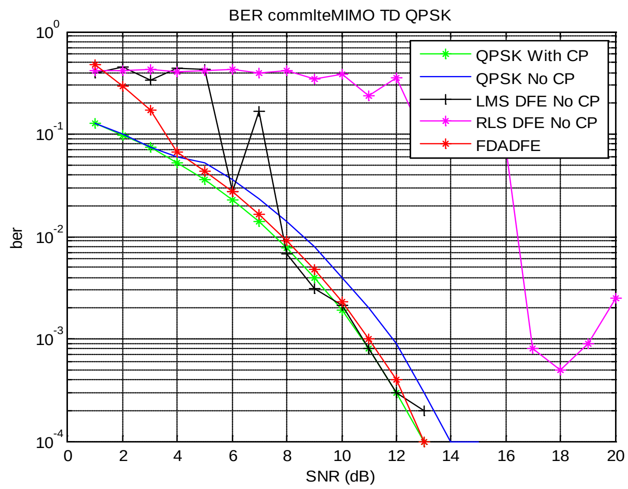

Fig. 2 BER vs. SNR for LTE MIMO OFDM system with DFE of different algorithms for QPSK Modulation

Fig. 2 shows the BER characteristics of MIMO LTE OFDM excluding CP with FDADFE for QPSK modulated signal. In this case, the FDADFE performs clearly better than other algorithms. It also performs very well than the conventional system. It can be shown from the fig. that, the BER of conventional system including CP floors at 10-4 at the SNR value of 13, whereas, the proposed system (excluding CP) floors at the same level at SNR of 12, which is approximately similar to the conventional system including CP. Therefore, the observation concludes that, the FDADFE performs very well with OFDM system excluding CP in terms of BER as well as by effectively optimizing the bandwidth of approximately 25% as the conventional system employs 25% of the bandwidth including CP.

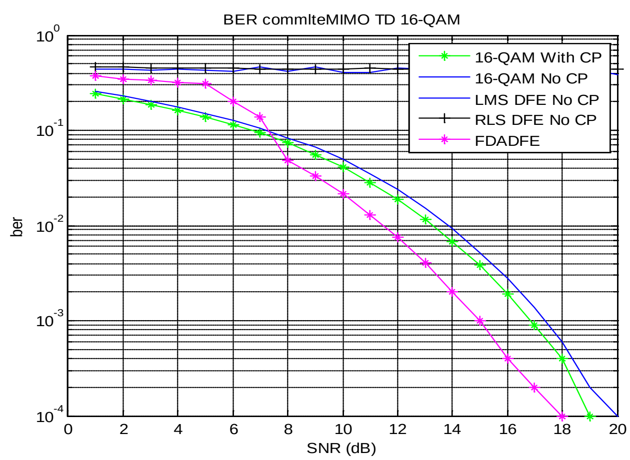

Fig. 3 BER vs. SNR for LTE MIMO OFDM system with DFE of different algorithms for16-QAM Modulation

Fig. 3 shows the SNR vs. BER performance characteristics of FDADFE for MIMO LTE OFDM system excluding CP for 16-QAM modulated signal. It can be observed that, the FDADFE performs lower to the conventional OFDM system in the range of 1-8dB. It is because the algorithms employed in the FDADFE cannot adapt the filter weights properly. But it outperforms the conventional system at high SNR values starting from 8dB. The comparison is done with the DFE of two other algorithms. A noteworthy observation in this fig. reveals that, the FDADFE for OFDM system excluding CP outperforms the conventional system for both including and excluding CP in terms of BER but effectively increase in utilizing the bandwidth of approximately 25% as the conventional system employs 25% of the bandwidth using CP. This surveillance suggests that the FDADFE is extremely competent for the effective use of the dynamic path diversity of the wireless channel.

Fig. 4 shows the SNR vs. BER performance of MIMO LTE OFDM system excluding CP with FDADFE for 64-QAM modulated signal. In this case, the FDADFE performs negligible to the conventional system in the range of SNR value of 1-13dB. This is also the reason for slower adaptation of filter weights as it uses 6 bits to modulate a symbol. But the performance is approximately same as the conventional system at high SNR values above 18dB. So, it can be noted that, the FDADFE with OFDM system excluding CP performs the same as the conventional system but using the whole bandwidth. So, approximately 25% of the bandwidth is effectively utilized with the same BER performance. This observation suggests that the FDADFE is very efficient for the use of the path diversity of the wireless channel. This is exactly the motivation why an LTE OFDM system with the FDADFE outperforms the conventional system with the utilized bandwidth at its maximum level.

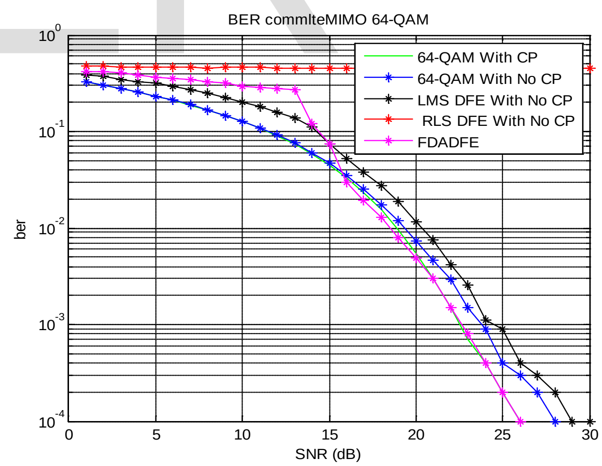

Fig. 4 BER vs. SNR for LTE MIMO OFDM system with FDADFE and conventional system for 64-QAM Modulation

4.2 Low power Consumption with FDADFE for LTE MIMO OFDM system excluding CP

This subsection analyses the power consumption of FDADFE vs. conventional LTE OFDM system excluding CP in terms of PAPR with CCDF measurement.

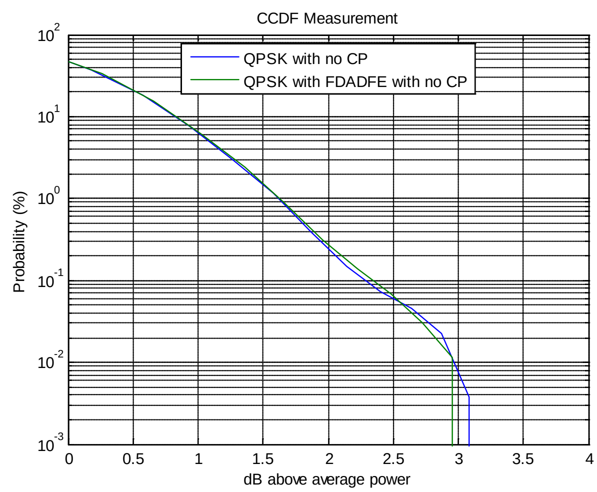

Fig. 5 PAPR for the conventional and proposed OFDM system excluding CP for QPSK modulation

Fig. 5 shows the comparison of PAPR for the conventional LTE system and the LTE MIMO OFDM system excluding CP for QPSK Modulation for SNR 20.

Fig. 6 provides the comparison of PAPR for the conventional LTE system and the LTE MIMO OFDM system excluding CP for 16-QAM Modulation for SNR 20.

Fig. 7 discovers the comparison of PAPR for the conventional LTE system and the LTE MIMO OFDM system excluding CP for 64-QAM Modulation for SNR 30.

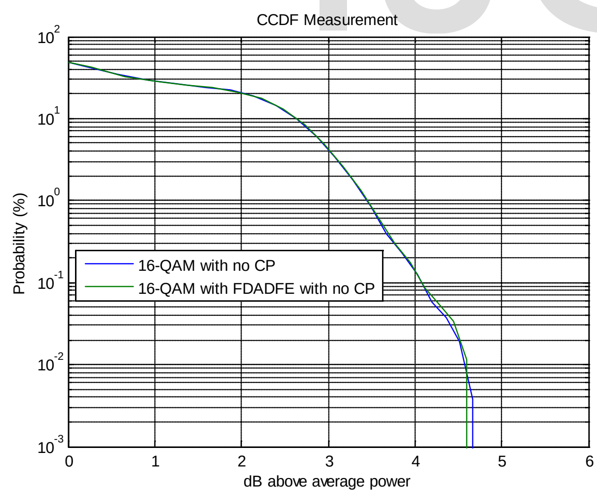

Fig. 6 PAPR for the conventional and proposed OFDM system excluding CP for 16-QAM modulation

Fig. 5-7 prove the same apprehension that, although the proposed FDADFE experiences more computational complexity, it does not consume much power than the conventional OFDM system excluding CP. These observations suggest that the proposed FDADFE with LTE MIMO OFDM system excluding CP is highly competent in terms of low power consumption.

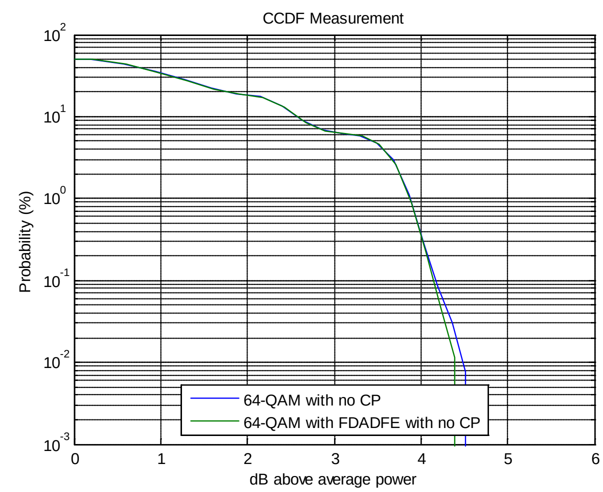

Fig. 7 PAPR for the conventional and proposed OFDM system excluding CP for 64-QAM modulation

5 Conclusion

In this paper, An adaptive algorithm based Decision Feedback Equalizer (FDADFE) is proposed for LTE MIMO OFDM system excluding CP with both the feed-forward and feedback filters operating in the frequency domain. Two adaptive algorithms (RLS and LMS algorithms) are applied to update the filter weights effectively for the FDADFE. For a certain combination of both filter coefficients, steady BER characteristics are obtained with QPSK and 16QAM modulation techniques. The 64-QAM performs same as the conventional system in terms of BER, whereas it performs better in case of bandwidth utilization. Further research is needed to improve the performance in this case. However, more power are saved for all three cases also through CCDF measurements.

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